Supplement of

Flow rate and source reservoir identification from airborne chemical sampling of the uncontrolled Elgin platform gas release

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Error analysis:

Method 1:

To calculate the error for method 1 we have

\[ \ln C_z = \ln C_0 - \frac{z^2}{2\sigma^2} \]  

(Eq S1)

From the observations, the gradient is determined and then \( q \) found using the value of \( C_0 \) and \( \sigma_z \) obtained from the intercept and gradient, i.e. \( q = f(\sigma_z, C_0) \). However, \( C_0 \) and \( \sigma_z \) are not truly independent, since in the majority of cases in (8) a change in \( C_0 \) is accompanied by a change in \( \sigma_z \) (if there is some constraint on the root-mean square error, RMSE, of the functional fit to the data). This functional relation of \( C_0 \) and \( \sigma_z \) suggests that traditional methods of error analysis are not appropriate in this case, and so we adopt a Monte-Carlo-simulation approach to determine the uncertainty in \( q \). The method is as follows.

We denote the intercept of the line of best-fit in (8) as \( \ln C_0 = \gamma \), with standard error \( E_{\gamma} \) and denote the gradient \((-1/2\sigma^2) = \mu \) with standard error \( E_{\mu} \). A large number (~5000) of unique straight-line fits were constructed, based upon \( \gamma, \mu, E_{\gamma} \) and \( E_{\mu} \). We then calculate the mean RMSE (equivalent to the RMSE of the line of best fit) and standard deviation \( \sigma_{RMSE} \) of the RMSE for all of these fits and selected a large number (~1000) of these such that the RMSE of each individual fit is less than \( \overline{RMSE} + \sigma_{RMSE} \) (this avoids combinations of intercept and gradient that lead to large RMSE values and would be considered poor fits to the data). From each line of best-fit, calculate \( \sigma_{z_k} \) and \( \gamma_k \) \((k = 1, \ldots, k_{max}; k_{max} \sim 1000)\).

The remaining parameters are sampled as follows. (i) We constructed a series of \( \sigma_{y_i} \) \((i = 1, \ldots, i_{max}; i_{max} \sim 1000)\)

\[ \sigma_{y_i} = \overline{\sigma_y} + \sigma_{y_i}' \]  

(Eq S2)

The noise \( \sigma_{y_i}' \) is produced artificially from normally distributed random numbers and has zero mean, with a standard deviation corresponding to the standard error of the observed data; \( \overline{\sigma_y} \) is the mean of the observed data.

(ii) Similarly, we constructed a sample of wind speeds \( U_j \) \((j = 1, \ldots, j_{max}; j_{max} \sim 1000)\)

\[ U_j = \overline{U} + U_j' \]  

(Eq S3)

Where, again, the noise \( U_j' \) is normally distributed noise with zero mean and standard deviation equal to the standard error of the observed data and \( \overline{U} \) is the mean of the observed data (the observations here referring to the appropriate flight legs). The assumption is made here that the wind speed \( U_j \) and dispersion parameter \( \sigma_{y_i} \) are distributed normally; more sophisticated assumptions, such as that the wind speed obeys a Weibull-type distribution, would be possible but are not likely to affect significantly the results).

We then, computed the mean source of the \( i_{max}j_{max}k_{max} \) (typically \( 10^9 \)) reconstructed profiles:

\[ \overline{q} = \frac{\sum_{i=1}^{i_{max}} \sum_{j=1}^{j_{max}} \sum_{k=1}^{k_{max}} \pi \sigma_{y_i} \sigma_{z_k} U_j e^{\gamma_k}}{i_{max} j_{max} k_{max}} \]  

(Eq S4)
The standard deviation of \( q \) is then calculated, based upon the individual samples of \( q \) and the mean \( \bar{q} \), as calculated above.

**Method 2:**

To calculate the error for this method we have:

\[
q = \sqrt{2\pi} C_0 U \sigma_y H
\]  
(Eq S5)

Similar to the above, a series of sources is reconstructed. The wind speed \( U \) and dispersion parameter \( \sigma_y \) are sampled as for Method 1. In addition, a series of \( C_0 \) and \( H \) are produced:

\[
\begin{align*}
C_{0i} &= \bar{C}_0 + C_{0i}' \quad (i=1, \ldots, i_{\text{max}}) \\
H_l &= \bar{H} + H_l' \quad (l=1, \ldots, l_{\text{max}})
\end{align*}
\]  
(Eq S6, Eq S7)

where \( \bar{C}_0 \) and \( \bar{H} \) are the mean of the observed concentrations and mixing-layer heights, respectively (in practice, there is only one value of \( H \) observed.) The added noise \( C_{0i}' \) and \( H_l' \) is (as for Method 1) designed to have zero mean, and standard deviation equal to the observed variable. In the case of mixing-layer depth, this is taken to be 10% of the observed value, typically 100 m. Reconstructed sources are then taken to be

\[
q_{i,j,k,l} = \sqrt{2\pi} C_{0j} U_j \sigma_{yj} H_l
\]  
(Eq S8)

where the total number of samples \( i_{\text{max}} j_{\text{max}} k_{\text{max}} l_{\text{max}} \) is taken to be of the order of a billion. The mean and standard deviation are then calculated in the usual manner.
Figure S1: (a) radiosonde atmospheric profile taken from Ekofisk during the time of flight B688 (b) dropsonde atmospheric profile from flight B727.
Figure S2: NOAA 100 km global Sea Surface Temperature (data-set derived from 8 km resolution satellite images) for the period of the flights. Air temperatures from radiosoundings and dropsondes.