Supplement of

MISR research-aerosol-algorithm refinements for dark water retrievals

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Real Refractive Index \((n_r)\) Sensitivity Study

[Figures S1 – S5]

This section expands on the AOD retrieval sensitivity analysis for the real refractive index \((n_r)\), illustrated in Figure 6 of the main paper. It covers a range of particle sizes, and \(n_r\) as well as AOD values are varied systematically for the particles assumed in the algorithm comparison space.

For each figure, the simulated atmosphere contains single-mode particles having \(n_r\) and effective radius \((r_e)\) given at the top. Comparison-space particles, having varying \(n_r\), are defined above each panel; comparisons are made for three values of AOD and a range of geometries. The geographic placement of the plots is for illustration – over-water conditions are assumed everywhere, with the surface pressure prescribed as 1013.25 mb, and the surface wind speed set to 2.5 m/s.

**Parameter Space**

Simulated Atmosphere \(r_e\) values (\(\mu m\)): 0.12, 0.26, 0.57, 1.28, 2.80
Simulated Atmosphere \(AOD\) values: 0.1, 0.5, 2.0
Simulated Atmosphere \(n_r\) value: 1.45
Comparison Space \(n_r\) values: 1.35, 1.40, 1.50, 1.55
Input Atmospheric Particles: \( n_r = 1.45, r_e = 0.12 \mu m \)

**< +9% Discrepancy, Comp. space \( n_r = 1.35 \)**

Median % Deviation from Reference AOD, \( \text{Chsq}_{\text{chord}} \leq 1.00 \), 2011–06–20; Orbit = 61194, Path = 215 spherical_nonabsorbing_0.12_\( n_r = 1.35 \)

**< -4% Discrepancy, Comp. space \( n_r = 1.50 \)**

Median % Deviation from Reference AOD, \( \text{Chsq}_{\text{chord}} \leq 1.00 \), 2011–06–20; Orbit = 61194, Path = 215 spherical_nonabsorbing_0.12_\( n_r = 1.50 \)

**< +5% Discrepancy, Comp. space \( n_r = 1.40 \)**

Median % Deviation from Reference AOD, \( \text{Chsq}_{\text{chord}} \leq 1.00 \), 2011–06–20; Orbit = 61194, Path = 215 spherical_nonabsorbing_0.12_\( n_r = 1.40 \)

**< -8% Discrepancy, Comp. space \( n_r = 1.55 \)**

Median % Deviation from Reference AOD, \( \text{Chsq}_{\text{chord}} \leq 1.00 \), 2011–06–20; Orbit = 61194, Path = 215 spherical_nonabsorbing_0.12_\( n_r = 1.55 \)

Figure S1
Input Atmospheric Particles: $n_r=1.45$, $r_e=0.26\,\mu m$

< +32% Discrepancy, Comp. space $n_r=1.35$

< -13% Discrepancy, Comp. space $n_r=1.50$

< +19% Discrepancy, Comp. space $n_r=1.40$

< -27% Discrepancy, Comp. space $n_r=1.55$

Figure S2
Input Atmospheric Particles: $n_r=1.45$, $r_e=0.57 \mu m$

**< 41% Discrepancy, Comp. space $n_r=1.35$**

Median % Deviation from Reference AOD, $Chi_\text{square} \leq 1.00$: 2011–06–20; Orbit = 61194, Path = 215

**< +26% Discrepancy, Comp. space $n_r=1.40$**

Median % Deviation from Reference AOD, $Chi_\text{square} \leq 1.00$: 2011–06–20; Orbit = 61194, Path = 215

**< -17% Discrepancy, Comp. space $n_r=1.50$**

Median % Deviation from Reference AOD, $Chi_\text{square} \leq 1.00$: 2011–06–20; Orbit = 61194, Path = 215

**< -31% Discrepancy, Comp. space $n_r=1.55$**

Median % Deviation from Reference AOD, $Chi_\text{square} \leq 1.00$: 2011–06–20; Orbit = 61194, Path = 215

Figure S3
Input Atmospheric Particles: \( n_r = 1.45, \quad r_e = 1.28 \mu m \)

**No successful retrievals, Comp. space \( n_r = 1.35 \)**

Median % Deviation from Reference AOD, \( \chi^2 \) limit \( \leq 1.00 \); 2011–06–20; Orbit = 61194, Path = 215

**\(< +27\%\) Discrepancy, Comp. space \( n_r = 1.40 \)**

Median % Deviation from Reference AOD, \( \chi^2 \) limit \( \leq 1.00 \); 2011–06–20; Orbit = 61194, Path = 215

**\(< -9\%\) Discrepancy, Comp. space \( n_r = 1.50 \)**

Median % Deviation from Reference AOD, \( \chi^2 \) limit \( \leq 1.00 \); 2011–06–20; Orbit = 61194, Path = 215

**\(< -16\%\) Discrepancy, Comp. space \( n_r = 1.55 \)**

Median % Deviation from Reference AOD, \( \chi^2 \) limit \( \leq 1.00 \); 2011–06–20; Orbit = 61194, Path = 215

Figure S4
Input Atmospheric Particles: $n_r=1.45$, $r_e=2.80\ \mu m$

**No successful retrievals, Comp. space $n_r=1.35$**

For very large particles, if $n_r$ deviates by 0.05 or more no successful retrievals are obtained (i.e., some retrieval sensitivity to $n_r$)

**Few successful retrievals, Comp. space $n_r=1.40$**

**Few successful retrievals, Comp. space $n_r=1.50$**

Median % Deviation from Reference AOD, Chisq$_{red}$ ≤ 1.00: 2011–06–20; Orbit = 61194, Path = 215

**No successful retrievals, Comp. space $n_r=1.55$**

Median % Deviation from Reference AOD, Chisq$_{red}$ ≤ 1.00: 2011–06–20; Orbit = 61194, Path = 215

Figure S5
Real Refractive Index Sensitivity Study

Conclusions

[Figures S1 – S5]

• When $n_r$ is overestimated, AOD is systematically underestimated, and conversely. Generally, retrieved AOD values still fall within 0.05 or 20% AOD, except in extreme cases.

• Smaller particles are affected less by errors in $n_r$.

• Very large particles are so sensitive to changes in $n_r$ that mixtures might not pass the algorithm acceptance criteria if $n_r$ deviates too far (~0.05) from the correct value.

• Medium particles (0.26-0.57 µm) produce the largest AOD deviations for ~0.1 $n_r$ error, but are not sensitive enough to $n_r$ to retrieve the correct value. Summary:
  - $r_e = 0.12$ µm: 5%-7.5% max. deviation for every 0.1 deviation from the correct $n_r$ [Figure S1]
  - $r_e = 0.26$ µm: 20%-30% max. deviation for every 0.1 deviation from correct $n_r$ [Figure S2]
  - $r_e = 0.57$ µm: 20%-40% max. deviation for every 0.1 deviation from correct $n_r$ [Figure S3]
  - $r_e = 1.28$ µm: 15%-40% max. deviation for every 0.1 deviation from correct $n_r$ [Figure S4]
  - $r_e = 2.80$ µm: variable max. deviation for every 0.1 deviation from correct $n_r$ [Figure S5]

• Distributions having larger effective radii tend to have biases that vary considerably depending on viewing/solar geometry.

• Overall, the 0.57 µm particle tends to perform the worst if $n_r$ is incorrect; the 0.26 µm particle is a close second. (Mixtures might still pass, but the retrieved AOD discrepancy can be >0.05/20%).
This section expands on the AOD retrieval sensitivity analysis for Linear Mixing and Modified Linear Mixing, as illustrated in Figure 7 of the main paper. MLM is used to approximate the radiative effects of mixtures containing two or more optically distinct aerosol components, based on pre-run radiative transfer calculations for the components individually [Abdou et al., 1997]. This supplement considers bi-modal particle distributions covering a range of particle sizes and SSA values, comparing retrieved AOD results from LM and MLM with those derived from runs of the radiative transfer code using layer-effective phase function (Equs. 3 of the main text).

For each figure in this section, the simulated atmosphere and retrieval climatology contain the same bi-modal mixture of particles, specified in the figure annotation, and taken from the SA climatology [Kahn et al., 2010]. Comparisons are made between radiative transfer runs using layer-effective phase function and LM or MLM approximations, for five values of AOD and a range of geometries. (The geographic placement of the plots is for illustration – all retrievals are performed over simulated black surfaces.) Mixture numbers in the figures correspond to the climatology in the MISR V22 SA climatology.

[Note that MLM reduces to LM when all particles in the mixture are non-absorbing or have the same SSA.]
Impact of **Linear Mixing;** Globally, Non-Absorbing Aerosol

- **Linear mixing overestimates** AOD with spherical **non-absorbing** particles
- A larger particle **size difference** $\rightarrow$ larger AOD **overestimate**

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**Note:** At low lat, moderate AOD $\rightarrow$ No mixtures pass when LM is used

- < +17% Discrepancy, **Mix #5**
  - 70% sph_nonabs_0.06
  - 30% sph_nonabs_2.80

- < +16% Discrepancy, **Mix #9**
  - 30% sph_nonabs_0.06
  - 70% sph_nonabs_2.80

- < +18% Discrepancy, **Mix #7**
  - 50% sph_nonabs_0.06
  - 50% sph_nonabs_2.80

**Figure S6**

$0.06 + 2.80 \mu m$ $r_e$ particles
Impact of **Linear Mixing**: Globally, Non-Absorbing Aerosol

- Linear mixing **overestimates** AOD with spherical **non-absorbing** particles
- A larger particle size difference → larger AOD **overestimate**

**Note**: At low lat, moderate AOD → **No mixtures pass** when LM is used

< +7% Discrepancy, **Mix #15**

- 70% sph_nonabs_0.12
- 30% sph_nonabs_2.80

< +7% Discrepancy, **Mix #19**

- 30% sph_nonabs_0.12
- 70% sph_nonabs_2.80

< +7% Discrepancy, **Mix #17**

- 50% sph_nonabs_0.12
- 50% sph_nonabs_2.80

0.12 + 2.80 µm \( r_e \) particles

Figure S7
Impact of **Linear Mixing;** Globally, Non-Absorbing Aerosol

- **Linear mixing overestimates** AOD with spherical **non-absorbing** particles
- A larger particle **size difference** → larger AOD **overestimate**
Impact of **Linear Mixing**; Globally, Non-Absorbing Aerosol

- **Linear mixing overestimates** AOD with spherical **non-absorbing** particles
- A larger particle size difference → larger AOD **overestimate**

\[
\text{Median % Deviation from Reference AOD, } \chi^2 \leq 1.00; \text{ Orbit = 65179, Path = 133} \\
\text{sph\_nonabs\_0.57, sph\_nonabs\_2.80; mix = 70:30} \\
\text{< +2% Discrepancy} \\
\text{70% sph\_nonabs\_0.57} \\
\text{30% sph\_nonabs\_2.80} \\
\text{< +2% Discrepancy} \\
\text{50% sph\_nonabs\_0.57} \\
\text{50% sph\_nonabs\_2.80} \\
\text{0.57 + 2.80 \( \mu \text{m} \)} \\
\text{\( r_e \) particles}
\]
Impact of **Linear Mixing**: Globally, Non-Absorbing Aerosol

- Linear mixing **overestimates** AOD with spherical **non-absorbing** particles
- A larger particle **size difference** $\Rightarrow$ larger AOD **overestimate**

- Discrepancies:
  - < +1% Discrepancy
    - 70% sph_nonabs_1.28
    - 30% sph_nonabs_2.80
  - < +1% Discrepancy
    - 50% sph_nonabs_1.28
    - 50% sph_nonabs_2.80

- Particle size:
  - 1.28 + 2.80 $\mu$m $r_e$ particles

**Figure S10**
Impact of **MLM**; Globally, Absorbing Mixtures

- MLM **overestimates** AOD with spherical non-absorbing/absorbing particle mixtures

**Note:** At low lat & moderate AOD → **No mixtures pass** when MLM is used

- **< +25% Discrepancy**
  - 80% sph_abs_0.12_SSA_0.95
  - 20% sph_nonabs_2.80

- **< +13% Discrepancy**
  - 20% sph_abs_0.12_SSA_0.95
  - 80% sph_nonabs_2.80

- **< +29% Discrepancy**
  - 50% sph_abs_0.12_SSA_0.95
  - 50% sph_nonabs_2.80

0.12 SSA_0.95 + 2.80 μm $r_e$ particles

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**Figure S11**
**Impact of MLM:** Globally, Absorbing Mixtures

< +28% Discrepancy, **Mix #34**
80% sph_abs_0.12_SSA-0.90  
20% sph_nonabs_2.80

< +17% Discrepancy, **Mix #40**
20% sph_abs_0.12_SSA-0.90  
80% sph_nonabs_2.80

< +31% Discrepancy, **Mix #37**
50% sph_abs_0.12_SSA-0.90  
50% sph_nonabs_2.80

**Note:** At low lat & high AOD ⇒ No mixtures pass when MLM is used

- MLM overestimates AOD with spherical non-absorbing/absorbing particle mixtures

**Figure S12**
Impact of **MLM**; Globally, Absorbing Mixtures

- **< +27% Discrepancy, Mix #44**
  - 80% sph_abs_0.12_SSA-0.80
  - 20% sph_nonabs_2.80

- **< +12% Discrepancy, Mix #50**
  - 20% sph_abs_0.12_SSA-0.80
  - 80% sph_nonabs_2.80

- **< +29% Discrepancy, Mix #47**
  - 50% sph_abs_0.12_SSA-0.80
  - 50% sph_nonabs_2.80

**Note:** At high AOD → **No mixtures pass** when MLM is used

- MLM **overestimates** AOD with spherical non-absorbing/absorbing particle mixtures

Figure S13
Impact of **MLM;** Globally, Absorbing Mixtures

- **MLM overestimates** AOD with spherical non-absorbing/absorbing particle mixtures

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**Figure S14**

- **< +31% Discrepancy**
  - 80% sph_abs_0.12_SSA-0.75
  - 20% sph_nonabs_2.80

- **< +11% Discrepancy**
  - 20% sph_abs_0.12_SSA-0.75
  - 80% sph_nonabs_2.80

- **< +28% Discrepancy**
  - 50% sph_abs_0.12_SSA-0.75
  - 50% sph_nonabs_2.80

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**Note:** At high AOD → **No mixtures pass** when MLM is used

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0.12 SSA_0.75 + 2.80 µm \( r_e \) particles
Impact of **MLM**; Globally, Non-Spherical Mixtures

- **< +10% Discrepancy, Mix #52**
  - 48% sph_nonabs_0.12
  - 32% sph_nonabs_2.80
  - 20% Medium Dust Grains

- **< +6% Discrepancy, Mix #58**
  - 24% sph_nonabs_0.12
  - 16% sph_nonabs_2.80
  - 60% Medium Dust Grains

- **< +9% Discrepancy, Mix #55**
  - 36% sph_nonabs_0.12
  - 24% sph_nonabs_2.80
  - 40% Medium Dust Grains

• Modified linear mixing **overestimates** AOD with non-spherical particle mixtures

*Figure S15*
Impact of **MLM**; Globally, Non-Spherical Mixtures

- **Mix #65**
  - < +10% Discrepancy
  - 40% sph_nonabs_0.12
  - 24% Medium Dust Grains
  - 36% Coarse Dust Spheroids

- **Mix #73**
  - < +9% Discrepancy
  - 0% sph_nonabs_0.12
  - 40% Medium Dust Grains
  - 60% Coarse Dust Spheroids

- **Mix #69**
  - < +9% Discrepancy
  - 20% sph_nonabs_0.12
  - 32% Medium Dust Grains
  - 48% Coarse Dust Spheroids

**Note:** At high lat & high AOD → No mixtures pass when MLM is used

- Modified linear mixing overestimates AOD with non-spherical particle mixtures

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**Figure S16**
Linear Mixing (LM) & Modified-Linear Mixing (MLM) Sensitivity Study

**Conclusions**

- For spherical non-absorbing mixtures (Mixtures #1-30) \([LM, Figures S6-S10]\)
  - Retrieved AOD values still fall within 0.05 or 20% AOD
  - \(<5%\) bias for AOD\(\leq 1.0\) in all cases
  - Larger particle size difference \(\rightarrow\) larger AOD overestimate
    - So the largest bias is for \([0.06 + 2.80 \, \mu m]\) mixtures
  - The effect becomes more pronounced at high AODs (up to \(18%\) for AOD > 2)

- For spherical absorbing mixtures, AOD is also overestimated \([MLM, Figures S11-S14]\)
  - The mixtures considered here (#31-50) perform reasonably well at low AODs
    - [sph_abs_0.12_SSA-0.80 or 90 & + 2.80 \, \mu m non-abs.]
      - \(<10%\) bias for AOD\(\leq 1.0\)
  - Mixtures having larger fractions 2.80 \, \mu m non-abs. particles (#34-40 & 44-49) can fall outside of 0.05 or 20% AOD for AOD\(\geq 2.0\) for some geometries
    - Biases of 10-30% for AOD >2.0, even when SSA 0.12 \, \mu m = 0.95

- For non-spherical mixtures, AOD is also overestimated \([MLM, Figures S15-S16]\)
  - But \(<10%\) bias for all non-spherical mixtures in the MISR V22 SA climatology